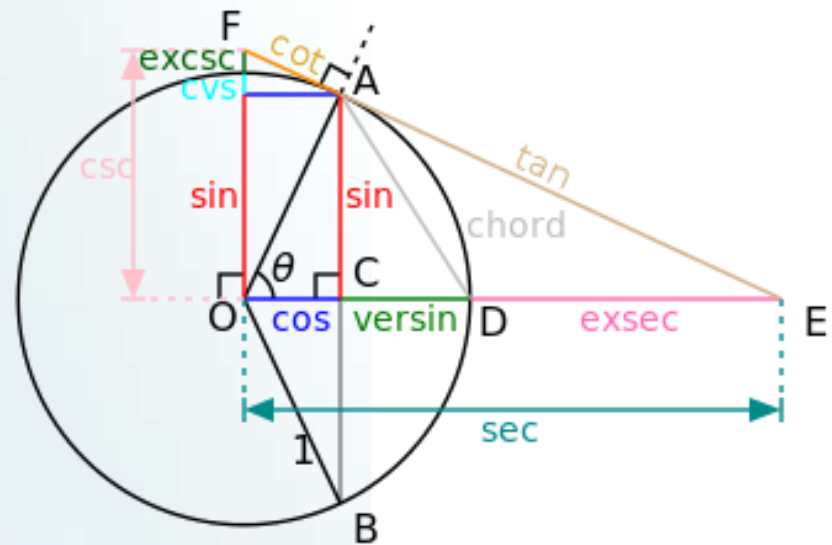


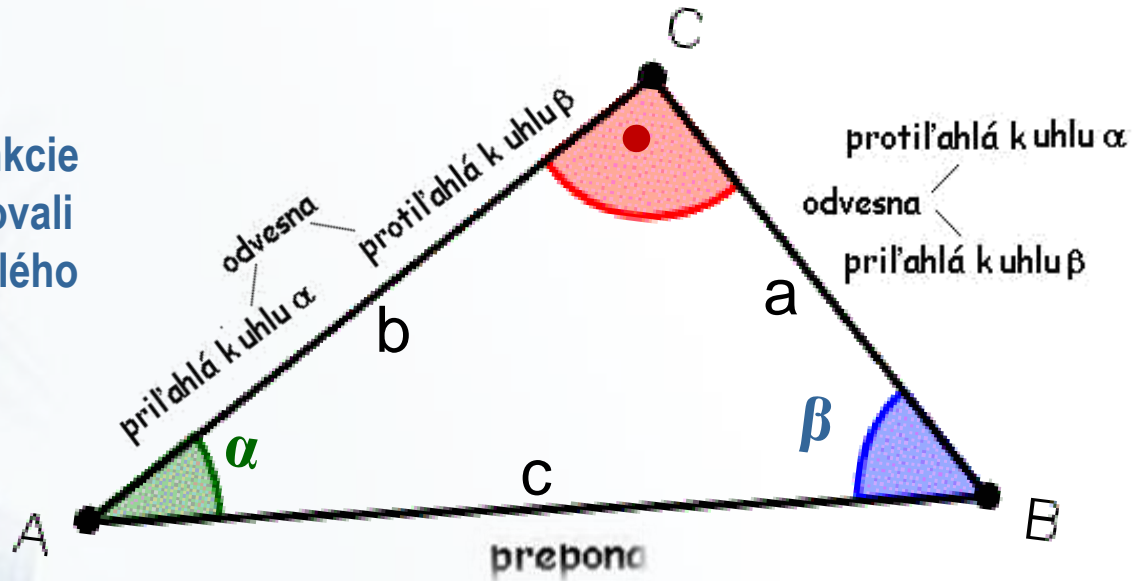
Goniometrické funkcie I

(úvod)



Goniometrické funkcie v pravouhlom trojuholníku

Goniometrické funkcie sa pôvodne definovali pomocou pravouhlého trojuholníka:



- \sin uhla je pomer proti'ahlej odvesny tohoto uhla a prepony:
- \cos uhla je pomer pril'ahlej odvesny k tomuto uhlu a prepony:
 - tg uhla je pomer proti'ahlej odvesny ku pril'ahlej:
 - cotg je pomer pril'ahlej odvesny ku proti'ahlej:

$$\sin \alpha = \frac{a}{c}$$

$$\cos \alpha = \frac{b}{c}$$

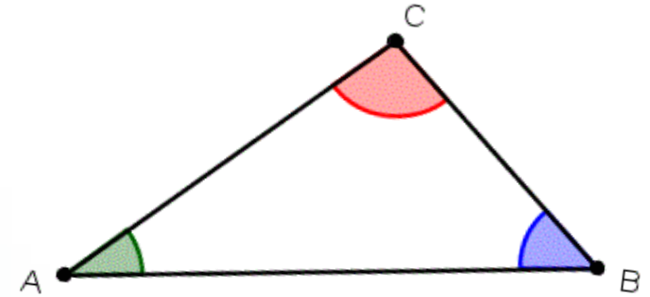
$$\operatorname{tg} \alpha = \frac{a}{b}$$

$$\operatorname{cotg} \alpha = \frac{b}{a}$$

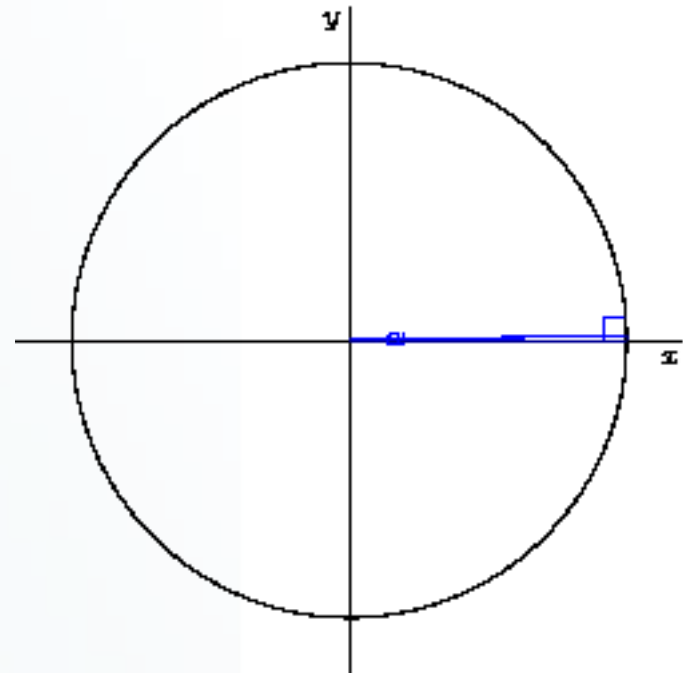
Goniometrické funkcie v pravouhlom trojuholníku

Definície goniometrických funkcií pomocou pravouhlého trojuholníka platia len pre uhly z intervalu $(0^\circ, 90^\circ)$.

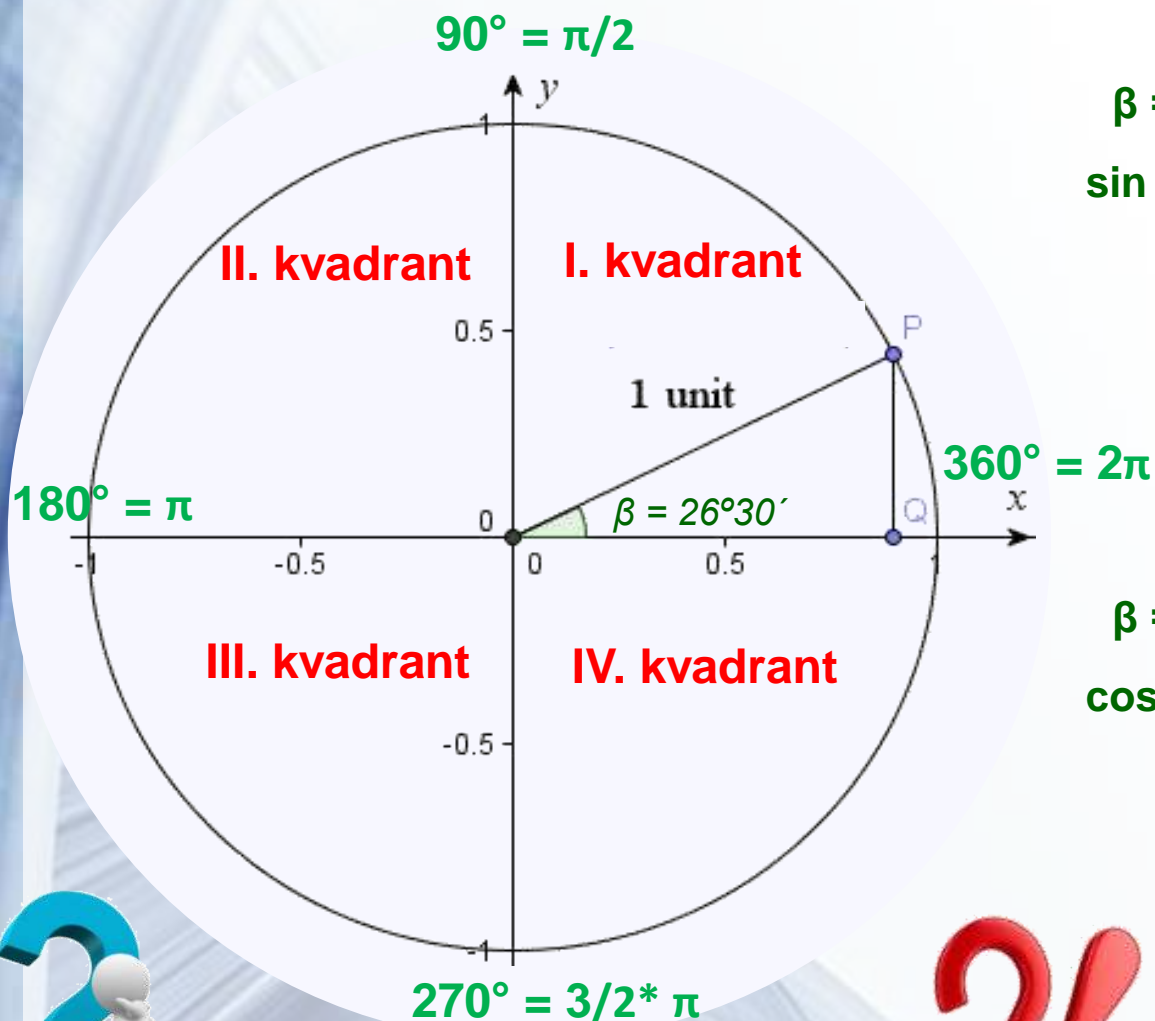
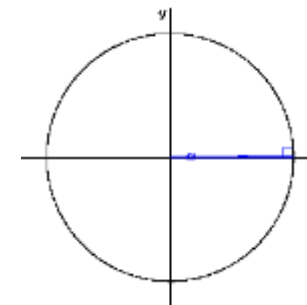
Dnes definujeme tieto funkcie pomocou jednotkovej kružnice.



Jednotková kružnica a pravouhlý trojuholník:



Sínus a kosínus uhla a jednotková kružnica:



$$\beta = 26^{\circ}30'$$

$$\sin \beta =$$



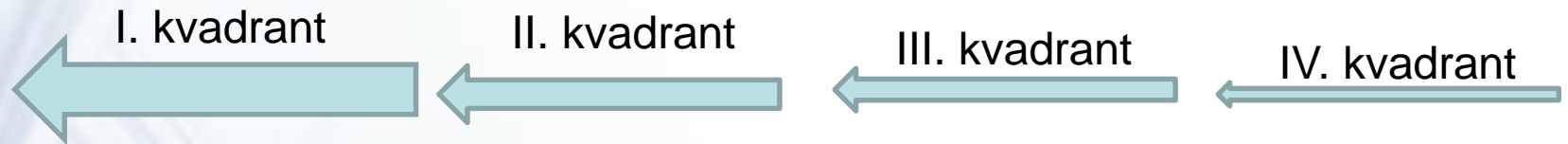
$$\beta = 26^{\circ}30'$$

$$\cos \beta =$$



...ako to spolu súvisí





Stup- ne	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
Radiá - ny	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$	2π

Prevod stupňov
na radiány:

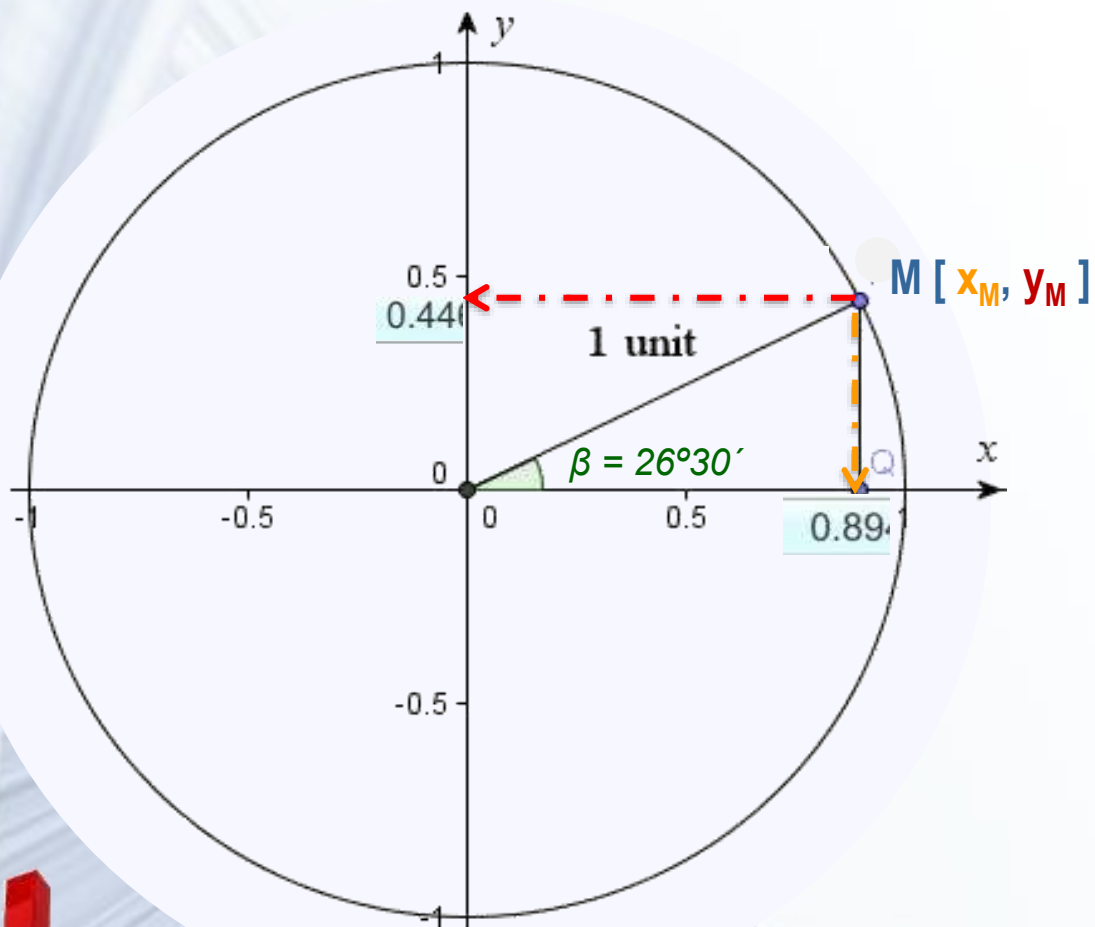
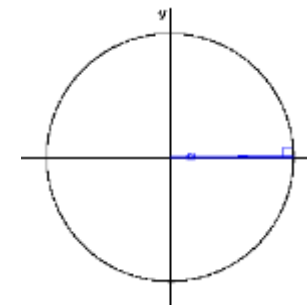
$$\alpha = x \cdot 180 / \pi$$

Prevod radiánov
na stupne :

$$x = \alpha \cdot \pi / 180$$

Stupne , radiány

Definície funkcií Sínus a Kosínus:



$$\beta = 26^{\circ}30'$$

$$\sin \beta = 0.44619781310981 = y_M$$

Sínus je funkcia, ktorá každému reálnemu číslu M zobrazenému na jednotkovej kružnici priradí druhú súradnicu (y) bodu M .

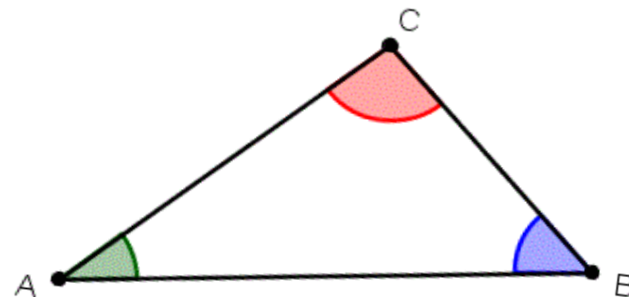
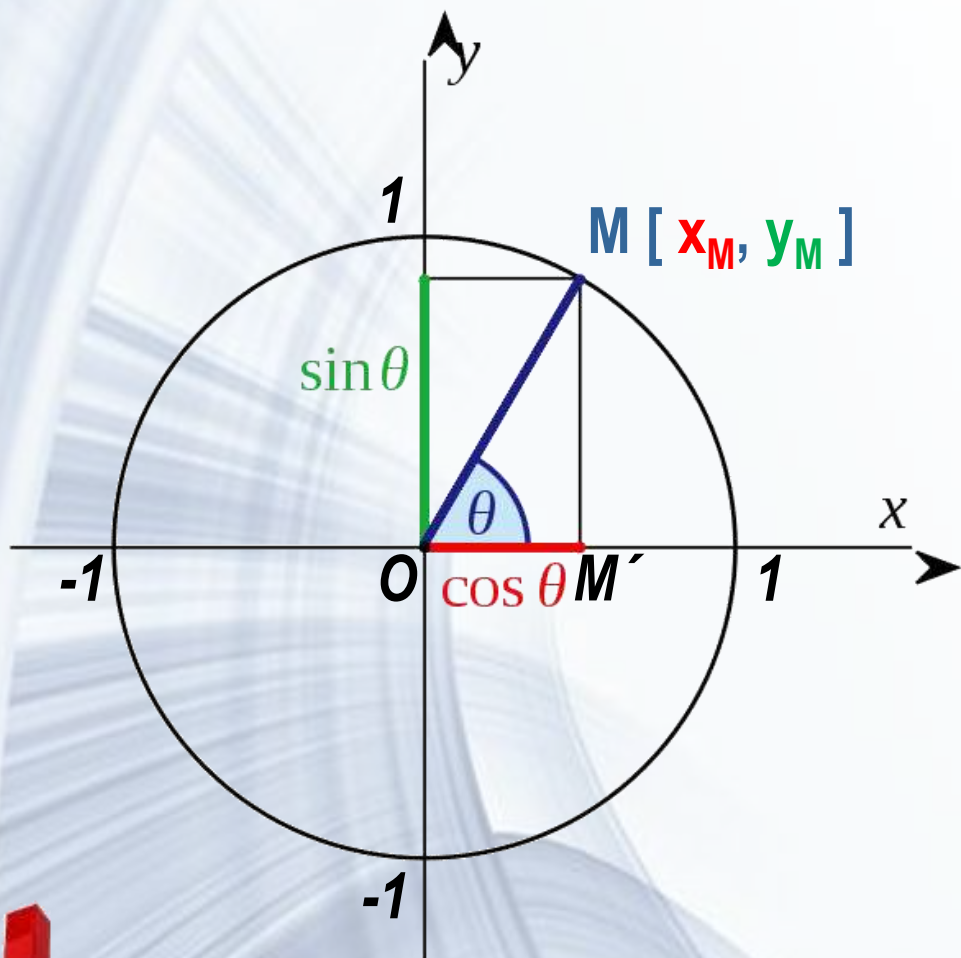
$$\beta = 26^{\circ}30'$$

$$\cos \beta = 0.89493436160203 = x_M$$

Kosínus je funkcia, ktorá každému reálnemu číslu M zobrazenému na jednotkovej kružnici priradí prvú súradnicu (x) bodu M .



Sínus, kosínus a jednotková kružnica – dôkaz:



Platí:

$$|OM| = 1$$
$$|OM'| = x_M$$
$$|MM'| = y_M$$

$$\sin \theta = \frac{|MM'|}{|OM|} = \frac{y_M}{1} = y_M$$

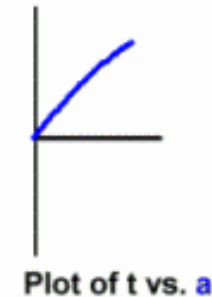
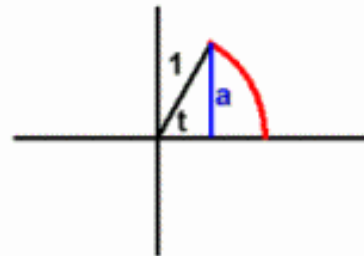
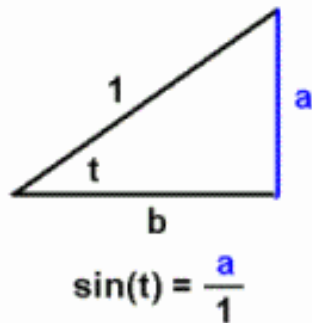
$$\cos \theta = \frac{|OM'|}{|OM|} = \frac{x_M}{1} = x_M$$



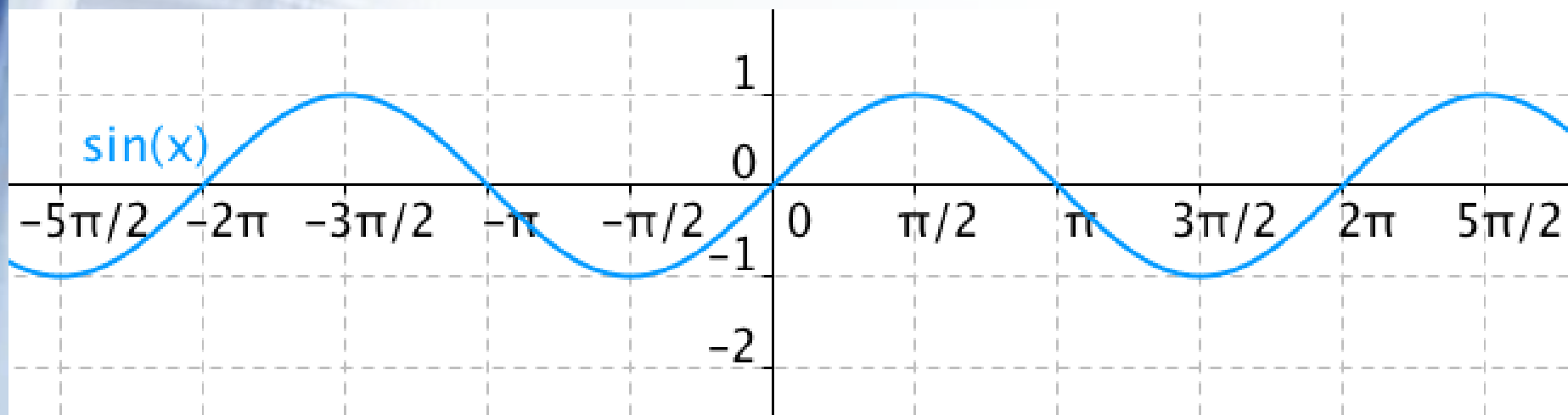
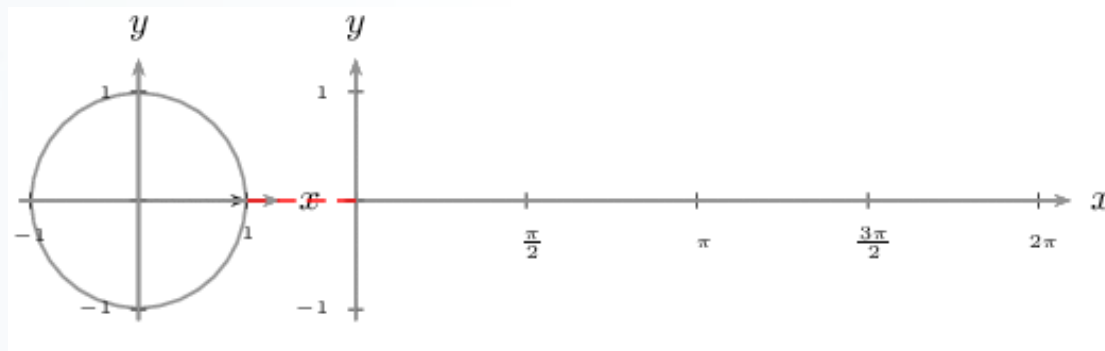
Jednotková kružnica a graf funkcie sínus:

Transformácia hodnôt súradníc $y = \sin \beta$ všetkých bodov jednotkovej kružnice na intervale $(0, 2\pi)$ do súradnicovej sústavy (animácia):

**Generation of the sine function
by "wrapping" around a unit circle.**



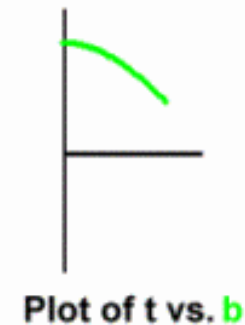
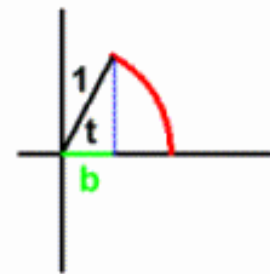
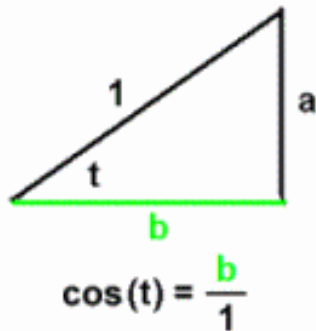
Grafom funkcie $y = \sin(x)$ je **sínusoida**:



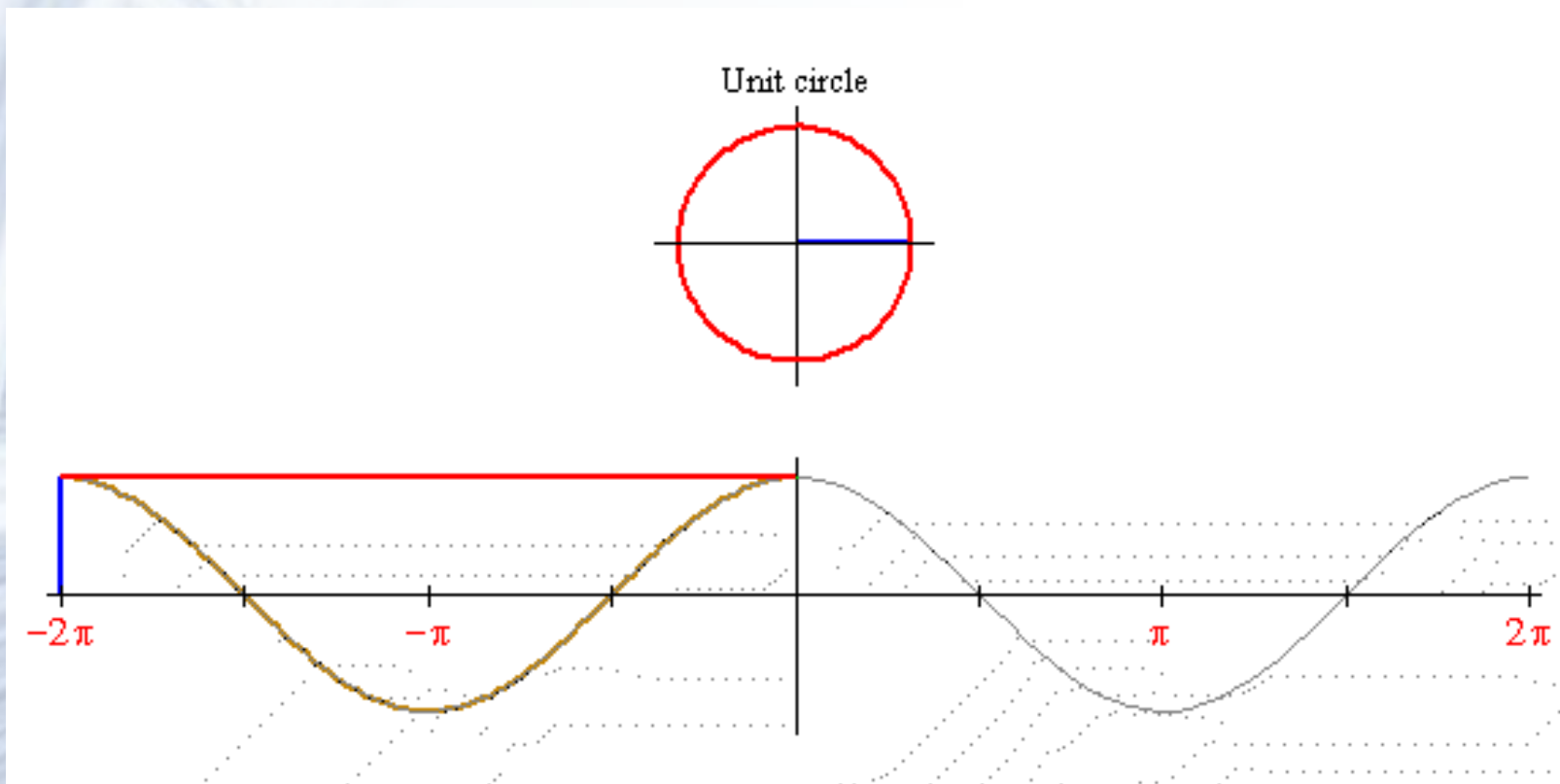
Jednotková kružnica a graf funkcie cos:

Transformácia hodnôt súradníc $x = \cos \beta$ všetkých bodov jednotkovej kružnice na intervale $(0, 2\pi)$ do súradnicovej sústavy:

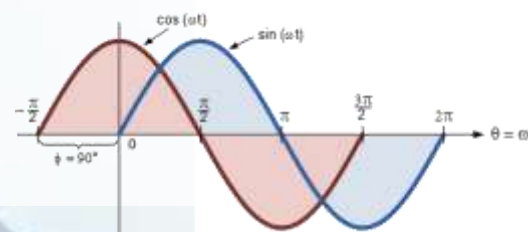
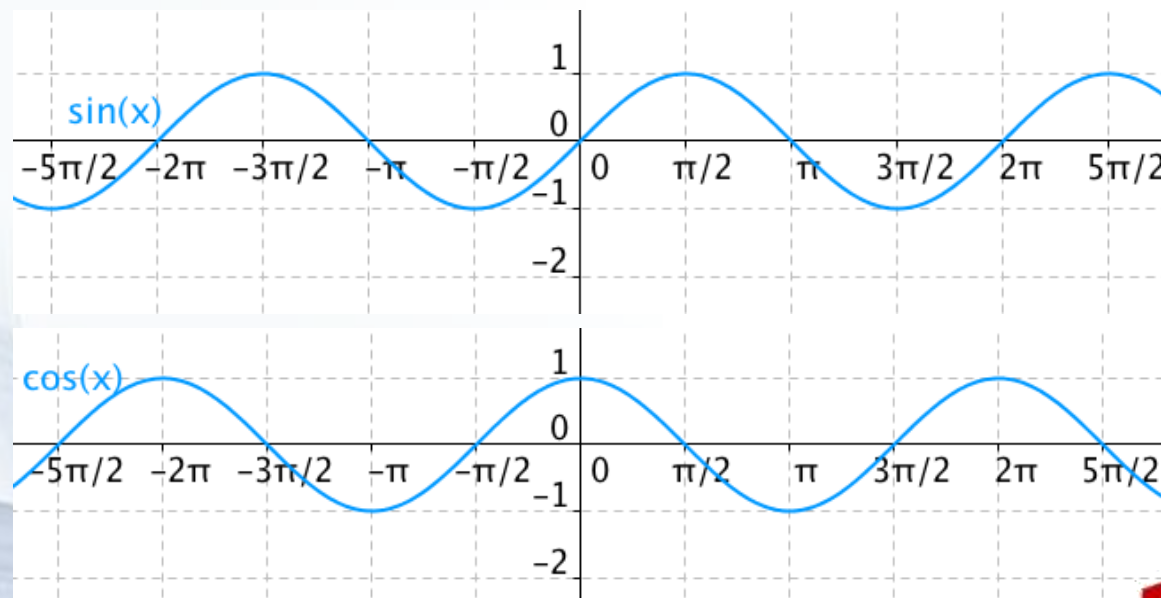
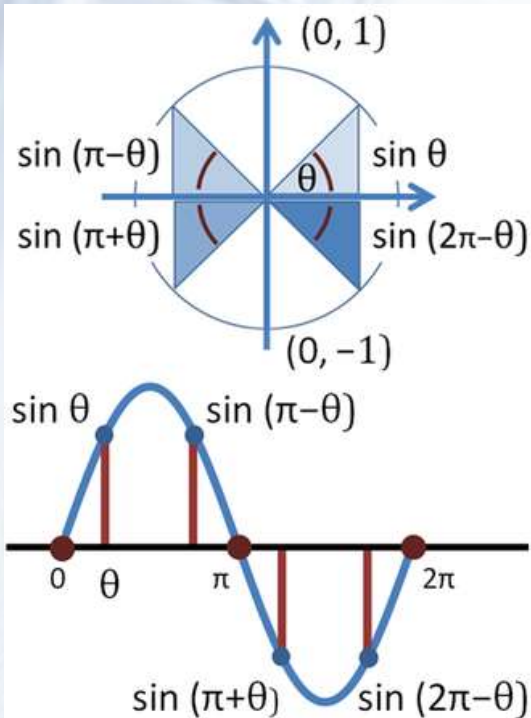
**Generation of the cosine function
by "wrapping" around a unit circle.**



Grafom funkcie $y = \cos(x)$ je **kosínusoida**:



Trigonometric functions	The angles or arcs x																
	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1



...niečo zaujímavé

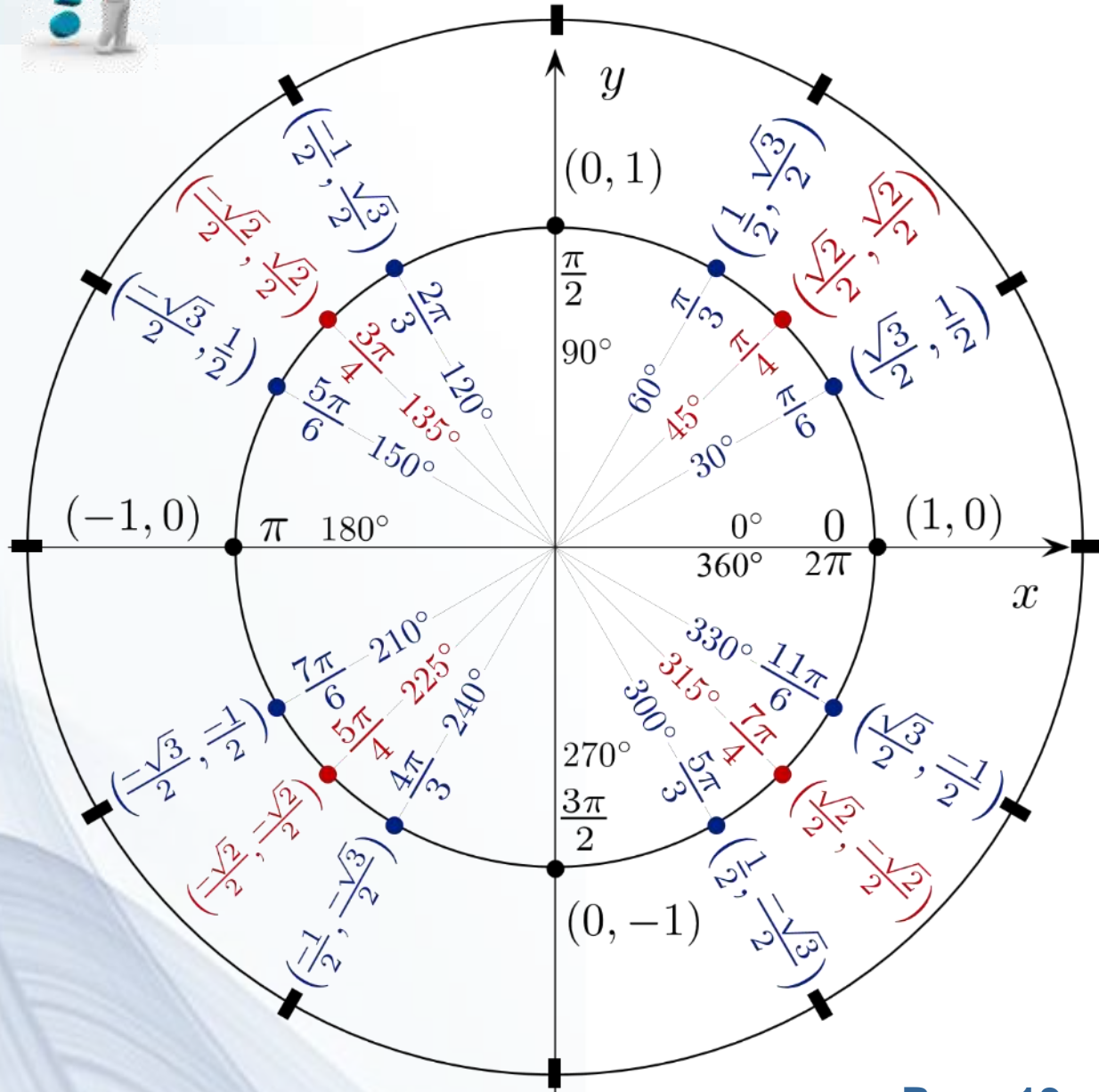
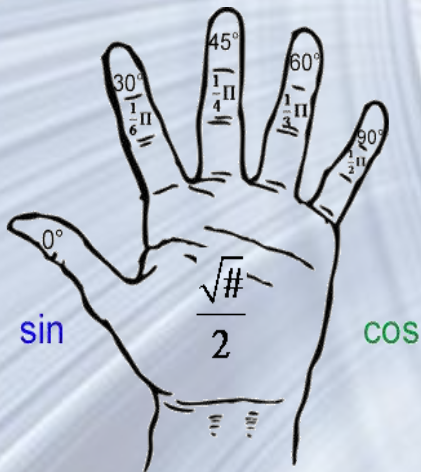


Ako si to zapamätám



	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
	0°	30°	45°	60°	90°
$\sin x$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
	0°	30°	45°	60°	90°
$\cos x$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$



Vlastnosti funkcie $y = \sin x$

- ▶ periodická, najm. perióda $p = 2\pi$, $\sin x = \sin(x + k \cdot 2\pi)$
- ▶ nie je prostá
- ▶ nepárna, $\sin(-x) = -\sin x$
- ▶ ohraničená $d = -1$, $h = 1$
- ▶ *minimum v bodoch* $\{ k \in \mathbb{Z}; \frac{3}{2}\pi + k \cdot 2\pi \}$
- ▶ *max. v bodoch* $\{ k \in \mathbb{Z}; \frac{\pi}{2} + k \cdot 2\pi \}$
- ▶ Rastúca na intervaloch $\left\langle -\frac{\pi}{2} + k \cdot 2\pi, \frac{\pi}{2} + k \cdot 2\pi \right\rangle$
- ▶ Klesajúca na intervaloch $\left\langle \frac{\pi}{2} + k \cdot 2\pi, \frac{3}{2}\pi + k \cdot 2\pi \right\rangle$

Vlastnosti funkcie $y = \cos x$

- ▶ periodická, najm. perióda $p = 2\pi$, $\cos x = \cos(x + k \cdot 2\pi)$
- ▶ nie je prostá
- ▶ párna, $\cos(-x) = \cos x$
- ▶ ohraničená $d = -1$, $h = 1$
- ▶ *minimum v bodoch* $\{k \in \mathbb{Z}; 0 \text{ rad} + k \cdot 2\pi\}$
- ▶ *max. v bodoch* $\{k \in \mathbb{Z}; \pi + k \cdot 2\pi\}$
- ▶ Rastúca na intervaloch $\langle \pi + k \cdot 2\pi, 2\pi + k \cdot 2\pi \rangle$
- ▶ Klesajúca na intervaloch $\langle 0 + k \cdot 2\pi, \pi + k \cdot 2\pi \rangle$

1. Ktoré z čísel -101 ; -2π ; $-0,55$; 0 ; 1 ; $\sqrt{3}$; $0,9\pi$; $2 \cdot 10^{10}$ patria do definičného oboru funkcie:

a) $y = \sin x$

všetky uvedené

b) $y = \cos x$

**čísla
všetky uvedené**



Cvičenia

2 / 4

2. Ktoré z čísel $-2 ; -0,55 ; 0 ; 0,7 ; 1 ; \sqrt{2}$
patria do oboru hodnôt funkcie:

a) $y = \sin x$ **$-0,55 ; 0 ; 0,7 ; 1$**

b) $y = \cos x$ **$-0,55 ; 0 ; 0,7 ; 1$**



Cvičenia

3 / 4

3. Pre ktoré $x \in \langle 0, 2\pi \rangle$ platí:

a) $\sin x = 0$ $x \in \{0, \pi, 2\pi\}$

b) $\cos x = \frac{1}{2}$ $x \in \left\{ \frac{\pi}{3}, \frac{5}{3}\pi \right\}$



Cvičenia

4 / 4

4. Porovnajte: a) $\sin 181^\circ$ $<$ 0

b) $\cos 120^\circ$ $<$ 0

c) $\sin 140^\circ$ $>$ $\sin 300^\circ$

d) $\cos \frac{\pi}{4}$ $=$ $\cos \frac{7}{4}\pi$

e) $\sin 25^\circ$ $<$ $\cos 25^\circ$

